2.3 — OLS Linear Regression

ECON 480 • Econometrics • Fall 2020

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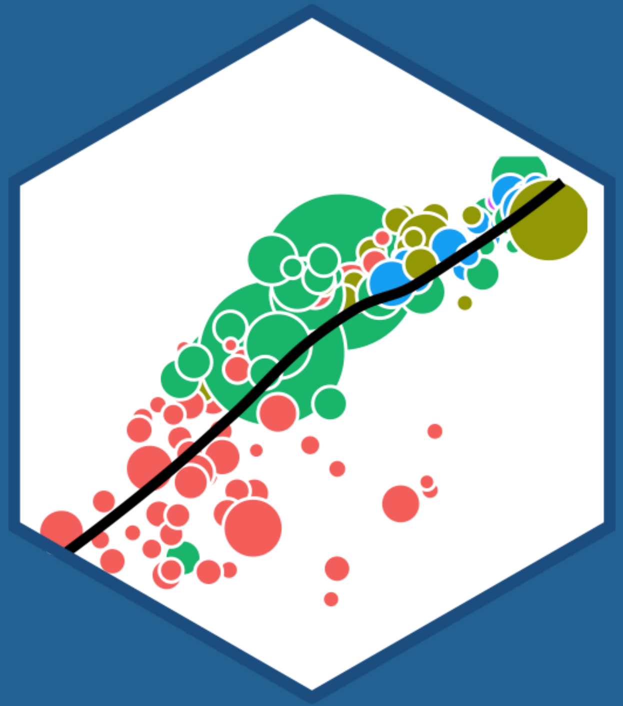


Exploring Relationships

Bivariate Data and Relationships

- We looked at single variables for descriptive statistics
- Most uses of statistics in economics and business investigate relationships *between* variables

Examples

- # of police & crime rates
- healthcare spending & life expectancy
- government spending & GDP growth
- carbon dioxide emissions & temperatures

Bivariate Data and Relationships

- We will begin with **bivariate** data for relationships between X and Y

- Immediate aim is to explore **associations** between variables, quantified with **correlation** and **linear regression**

- Later we want to develop more sophisticated tools to argue for **causation**

Bivariate Data: Spreadsheets I

```r
econfreedom <- read_csv("econfreedom.csv")
head(econfreedom)
```

```
## # A tibble: 6 x 6
##      X1 Country   ISO      ef    gdp continent
##   <dbl> <chr>     <chr> <dbl>  <dbl> <chr>
## 1     1 Albania   ALB    7.4   4543. Europe
## 2     2 Algeria   DZA    5.15  4784. Africa
## 3     3 Angola    AGO    5.08  4153. Africa
## 4     4 Argentina ARG    4.81 10502. Americas
## 5     5 Australia AUS    7.93 54688. Oceania
## 6     6 Austria   AUT    7.56 47604. Europe
```

- **Rows** are individual observations (countries)
- **Columns** are variables on all individuals

Bivariate Data: Spreadsheets II

```
econfreedom %>%
  glimpse()
```

```
## Rows: 112
## Columns: 6
## $ X1        <dbl> 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, …
## $ Country   <chr> "Albania", "Algeria", "Angola", "Argentina", "Australia", "…
## $ ISO       <chr> "ALB", "DZA", "AGO", "ARG", "AUS", "AUT", "BHR", "BGD", "BE…
## $ ef        <dbl> 7.40, 5.15, 5.08, 4.81, 7.93, 7.56, 7.60, 6.35, 7.51, 6.22,…
## $ gdp       <dbl> 4543.0880, 4784.1943, 4153.1463, 10501.6603, 54688.4459, 47…
## $ continent <chr> "Europe", "Africa", "Africa", "Americas", "Oceania", "Europ…
```

Bivariate Data: Spreadsheets III

```r
source("summaries.R") # use my summary_table function

econfreedom %>%
    summary_table(ef, gdp)
```

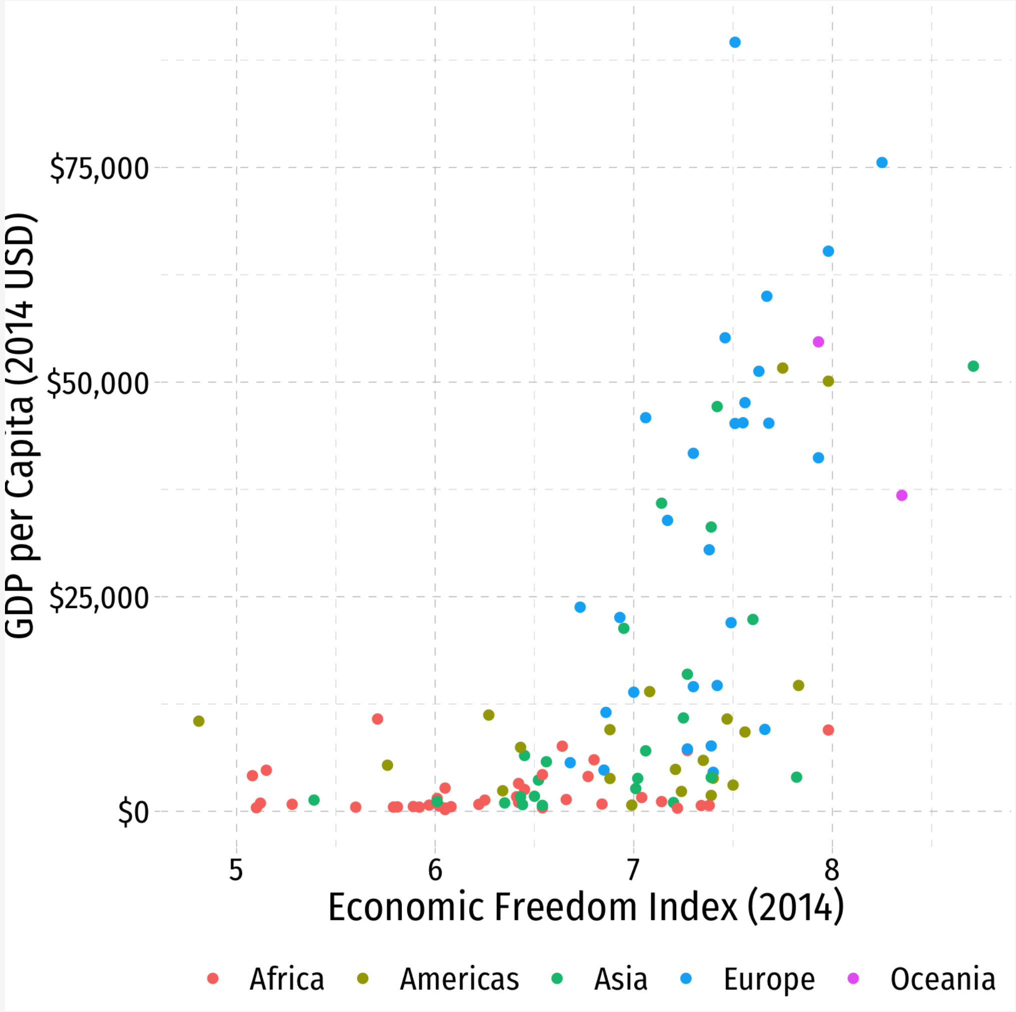
Variable	Obs	Min	Q1	Median	Q3	Max	Mean	Std. Dev.
ef	112	4.81	6.42	7.0	7.40	8.71	6.86	0.78
gdp	112	206.71	1307.46	5123.3	17302.66	89590.81	14488.49	19523.54

Bivariate Data: Scatterplots

- The best way to visualize an association between two variables is with a **scatterplot**

- Each point: pair of variable values $(x_i, y_i) \in X, Y$ for observation i

```r
library("ggplot2")
ggplot(data = econfreedom)+
  aes(x = ef,
      y = gdp)+
  geom_point(aes(color = continent),
             size = 2)+
  labs(x = "Economic Freedom Index (2014)",
       y = "GDP per Capita (2014 USD)",
       color = "")+
  scale_y_continuous(labels = scales::dollar)+
  theme_pander(base_family = "Fira Sans Condensed",
           base_size=20)+
  theme(legend.position = "bottom")
```



Associations

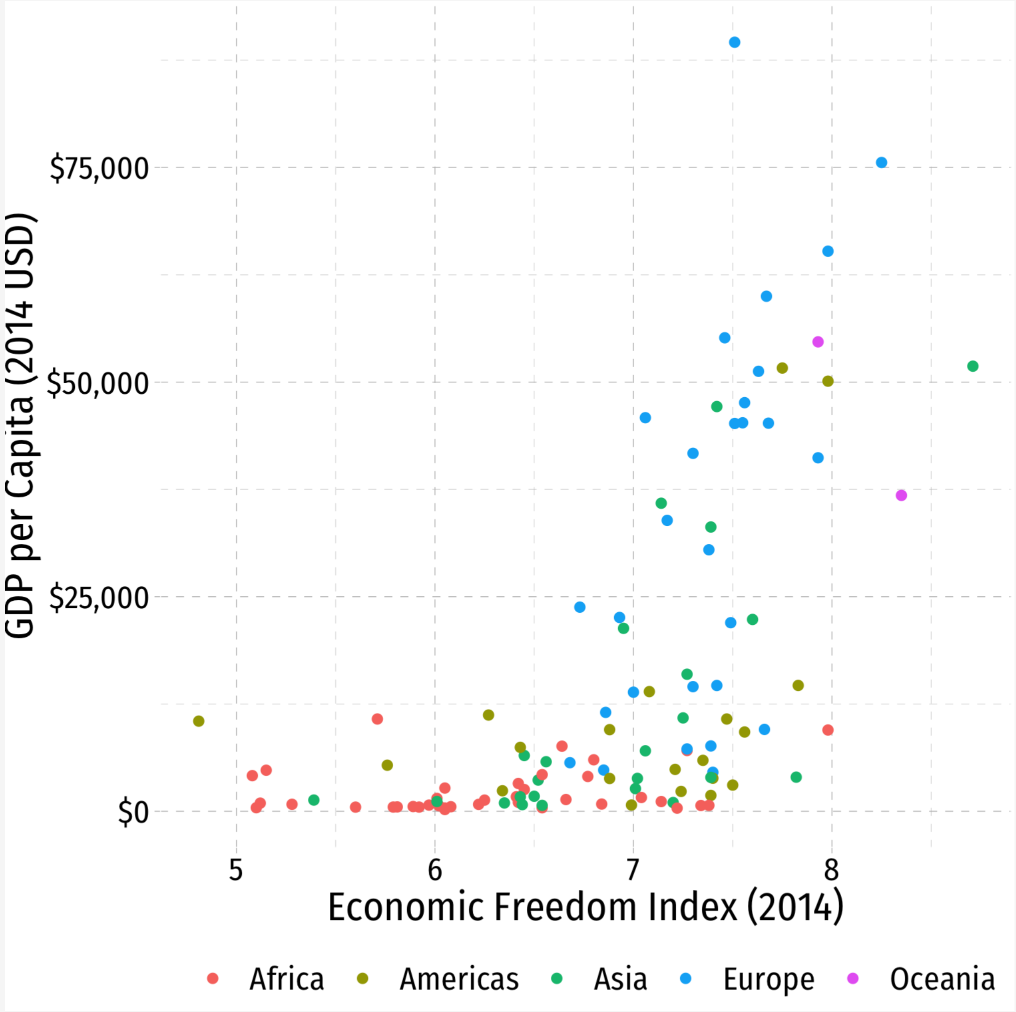
- Look for **association** between independent and dependent variables

1. **Direction**: is the trend positive or negative?

2. **Form**: is the trend linear, quadratic, something else, or no pattern?

3. **Strength**: is the association strong or weak?

4. **Outliers**: do any observations break the trends above?



Quantifying Relationships

Covariance

- For any two variables, we can measure their **sample covariance,** $cov(X, Y)$ or $s_{X,Y}$ to quantify how they vary *together*[†]

$$s_{X,Y} = E\big[(X - \bar{X})(Y - \bar{Y})\big]$$

- Intuition: if X is above its mean, would we expect Y:
  - to be *above* its mean also (X and Y covary *positively*)
  - to be *below* its mean (X and Y covary *negatively*)

- Covariance is a common measure, but the units are meaningless, thus we rarely need to use it so **don't worry about learning the formula**

[†] Henceforth we limit all measures to *samples*, for convenience. Population covariance is denoted $\sigma_{X,Y}$

Covariance, in R

```r
# base R
cov(econfreedom$ef,econfreedom$gdp)
```

```
## [1] 8922.933
```

```r
# dplyr

econfreedom %>%
  summarize(cov = cov(ef,gdp))
```

```
## # A tibble: 1 x 1
##     cov
##   <dbl>
## 1 8923.
```

8923 what, exactly?

Correlation

- More convenient to *standardize* covariance into a more intuitive concept: **correlation,** ρ or $r \in [-1, 1]$

$$r_{X,Y} = \frac{s_{X,Y}}{s_X s_Y} = \frac{cov(X,Y)}{sd(X)sd(Y)}$$

- Simply weight covariance by the product of the standard deviations of X and Y

- Alternatively, take the average[†] of the product of standardized (Z-scores for) each (x_i, y_i) pair:[‡]

$$r = \frac{1}{n-1} \sum_{i=1}^{n} \left(\frac{x_i - \bar{X}}{s_X} \right) \left(\frac{y_i - \bar{Y}}{s_Y} \right)$$

$$r = \frac{1}{n-1} \sum_{i=1}^{n} Z_X Z_Y$$

[†] Over n-1, a *sample* statistic!

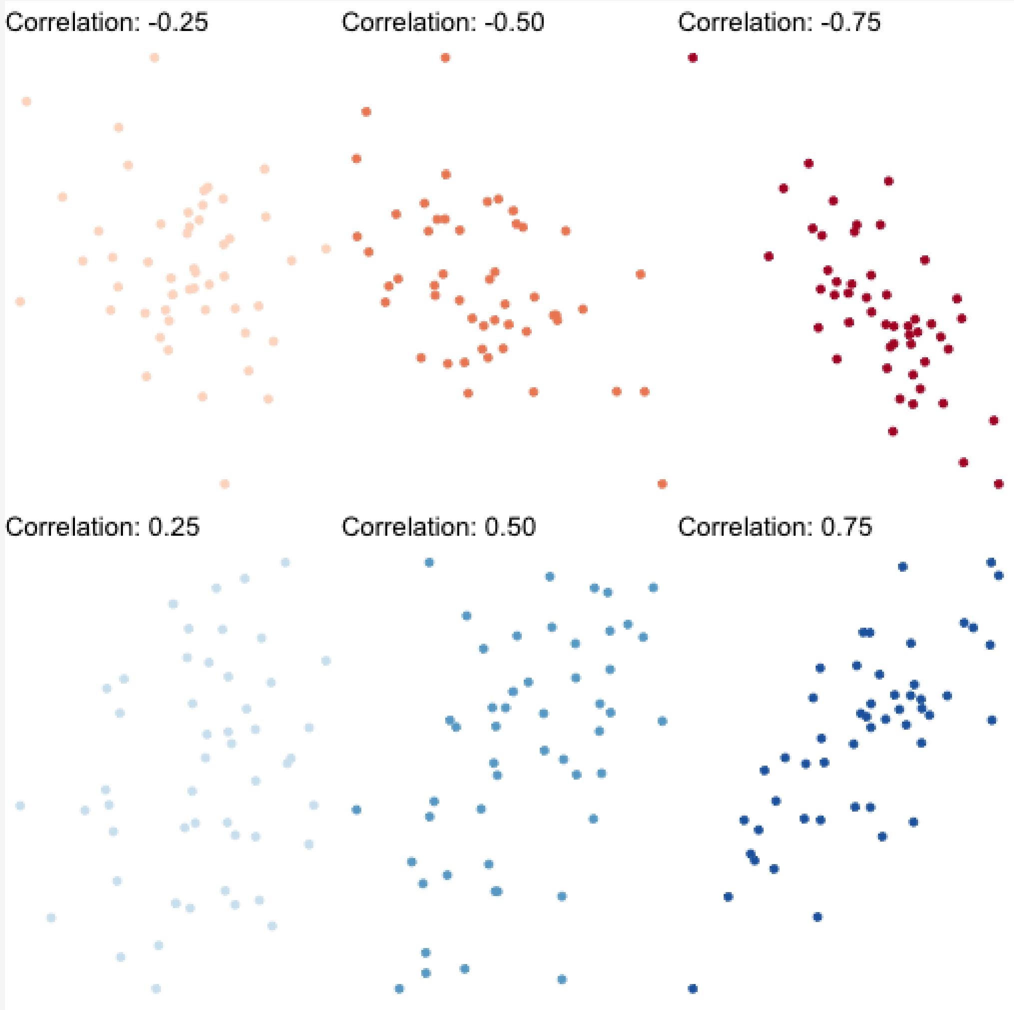
[‡] See today's class notes page for example code to calculate correlation "by hand" in R using the second method.

Correlation: Interpretation

- Correlation is standardized to

$$-1 \leq r \leq 1$$

  - Negative values $\implies$ negative association
  - Positive values $\implies$ positive association
  - Correlation of 0 $\implies$ no association
  - As $|r| \to 1$ $\implies$ the stronger the association
  - Correlation of $|r| = 1$ $\implies$ perfectly linear



Guess the Correlation!

[Guess the Correlation Game]

Correlation and Covariance in R

```r
# Base r: cov or cor(df$x, df$y)

cov(econfreedom$ef, econfreedom$gdp)
```

```
## [1] 8922.933
```

```r
cor(econfreedom$ef, econfreedom$gdp)
```

```
## [1] 0.5867018
```

```r
# tidyverse method

econfreedom %>%
  summarize(covariance = cov(ef, gdp),
            correlation = cor(ef, gdp))
```

```
## # A tibble: 1 x 2
##   covariance correlation
##        <dbl>       <dbl>
## 1      8923.       0.587
```

Correlation and Covariance in R I

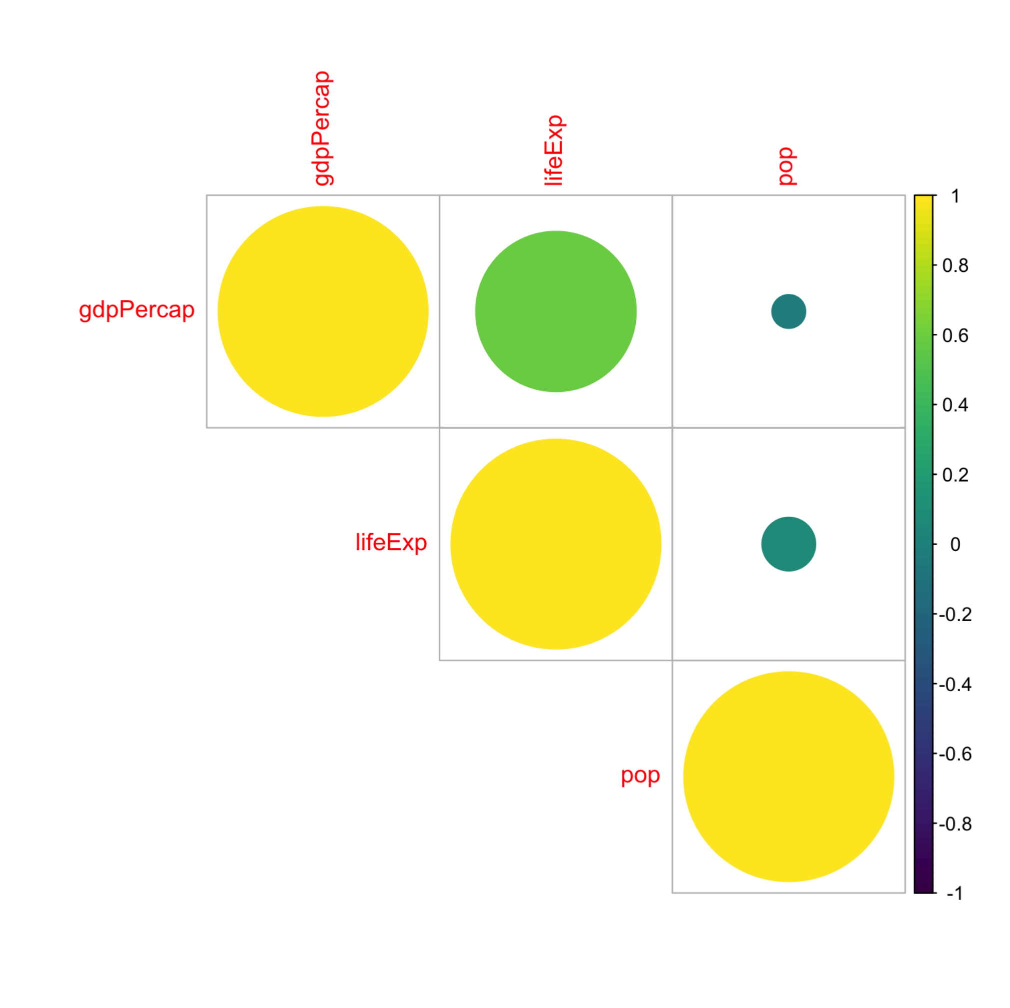
- `corrplot` is a great package (install and then load) to **visualize** correlations in data

```
library(corrplot) # see more at https://github.com/taiyun/corrplot
library(RColorBrewer) # for color scheme used here
library(gapminder) # for gapminder data

# need to make a corelation matrix with cor(); can only include numeric variables
gapminder_cor<- gapminder %>%
  dplyr::select(gdpPercap, pop, lifeExp)

# make a correlation table with cor (base R)
gapminder_cor_table<-cor(gapminder_cor)

# view it
gapminder_cor_table
```

```
##             gdpPercap         pop    lifeExp
## gdpPercap  1.00000000 -0.02559958 0.58370622
## pop       -0.02559958  1.00000000 0.06495537
## lifeExp    0.58370622  0.06495537 1.00000000
```

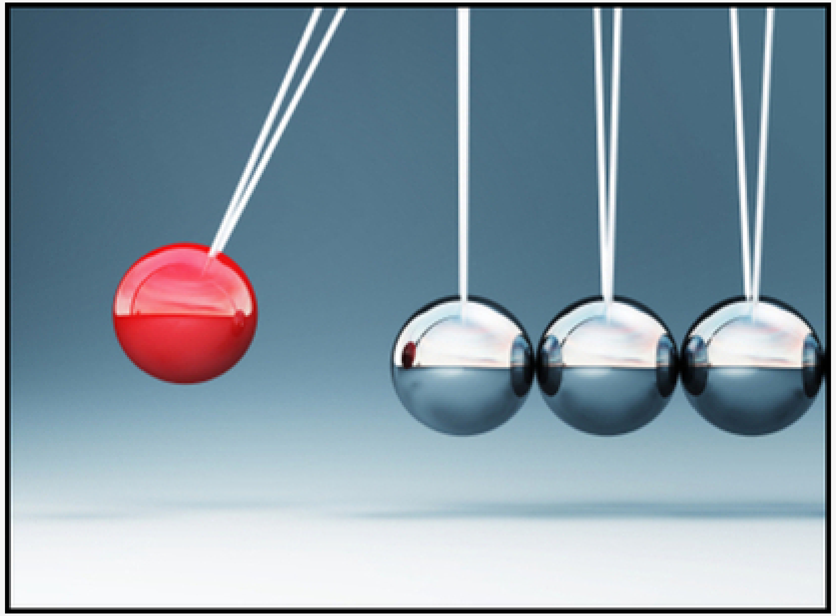
Correlation and Covariance in R II

```r
corrplot(gapminder_cor_table, type="upper",
         method = "circle",
         order = "alphabet",
         col = viridis::viridis(100)) # custom
```

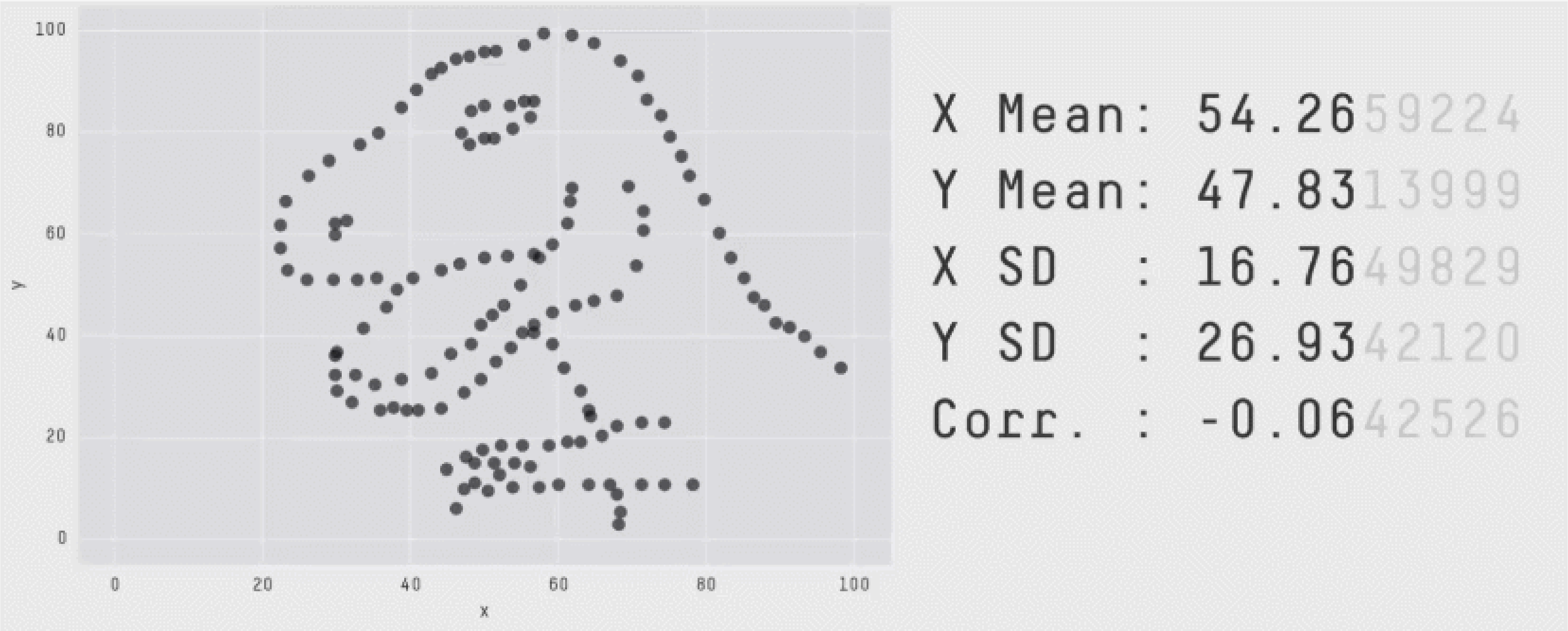


Correlation and Endogeneity

- Your Occasional Reminder: **Correlation does not imply causation!**
  - I'll show you the difference in a few weeks (when we can actually talk about causation)
- If X and Y are strongly correlated, X can still be **endogenous**!
- See today's class notes page for more on Covariance and Correlation



Always Plot Your Data!



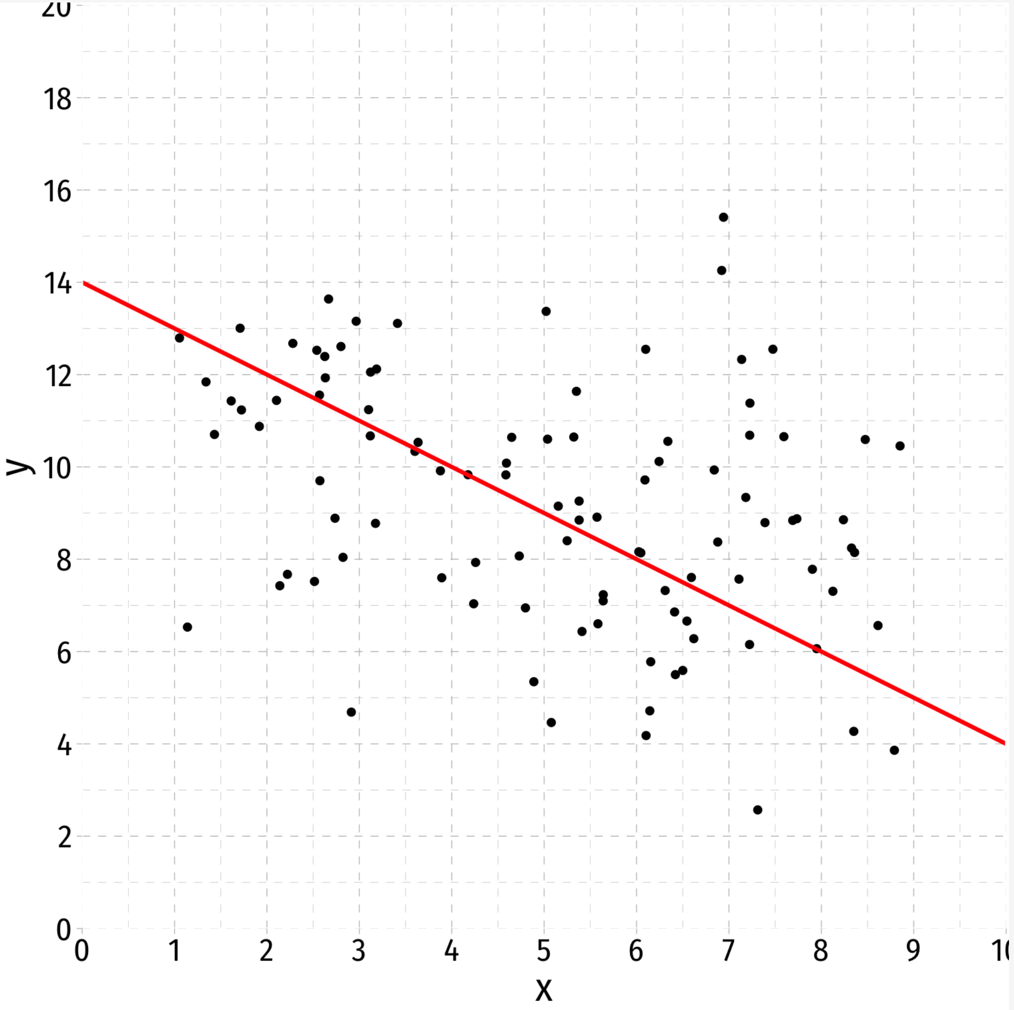
Linear Regression

Fitting a Line to Data

- If an association appears linear, we can estimate the equation of a line that would "fit" the data

$$Y = a + bX$$

- Recall a linear equation describing a line contains:.magenta
  - a: vertical intercept
  - b: slope



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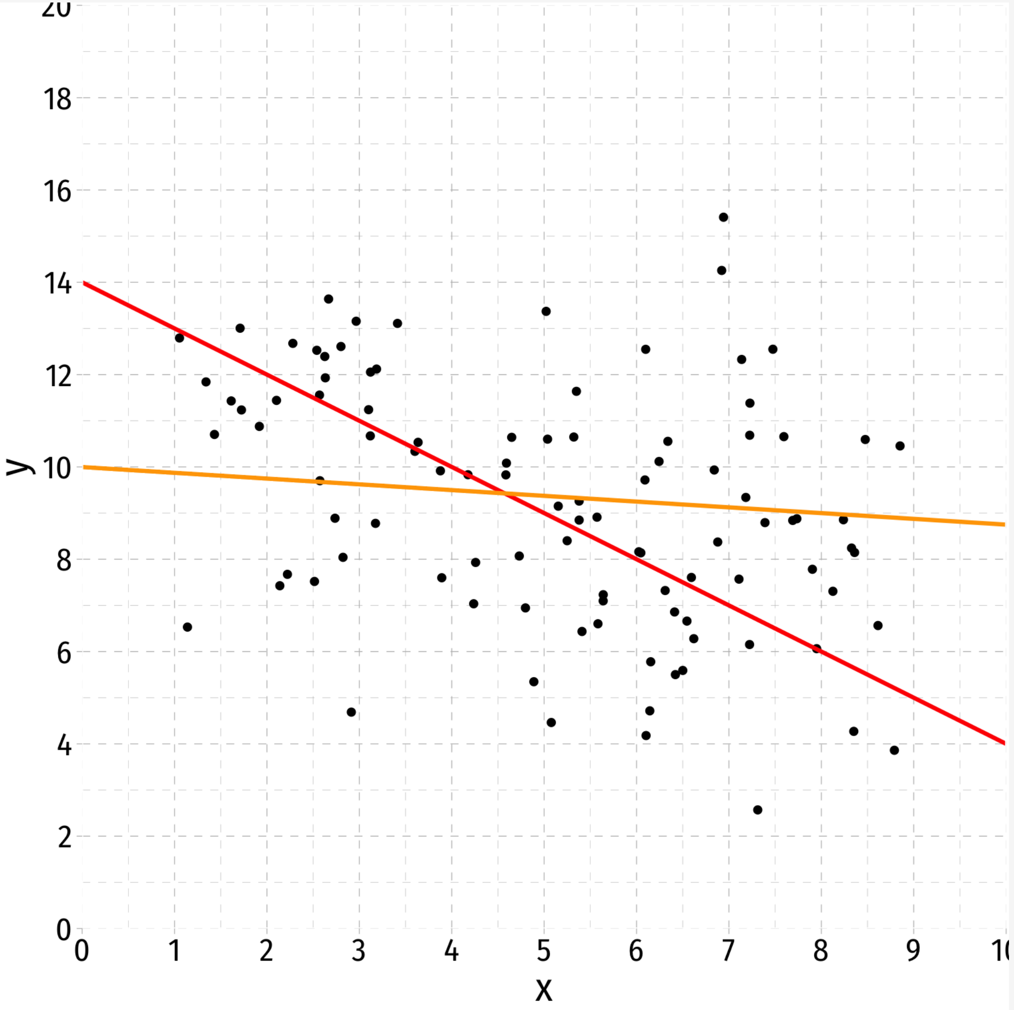
- How do we choose the equation that *best* fits the data?



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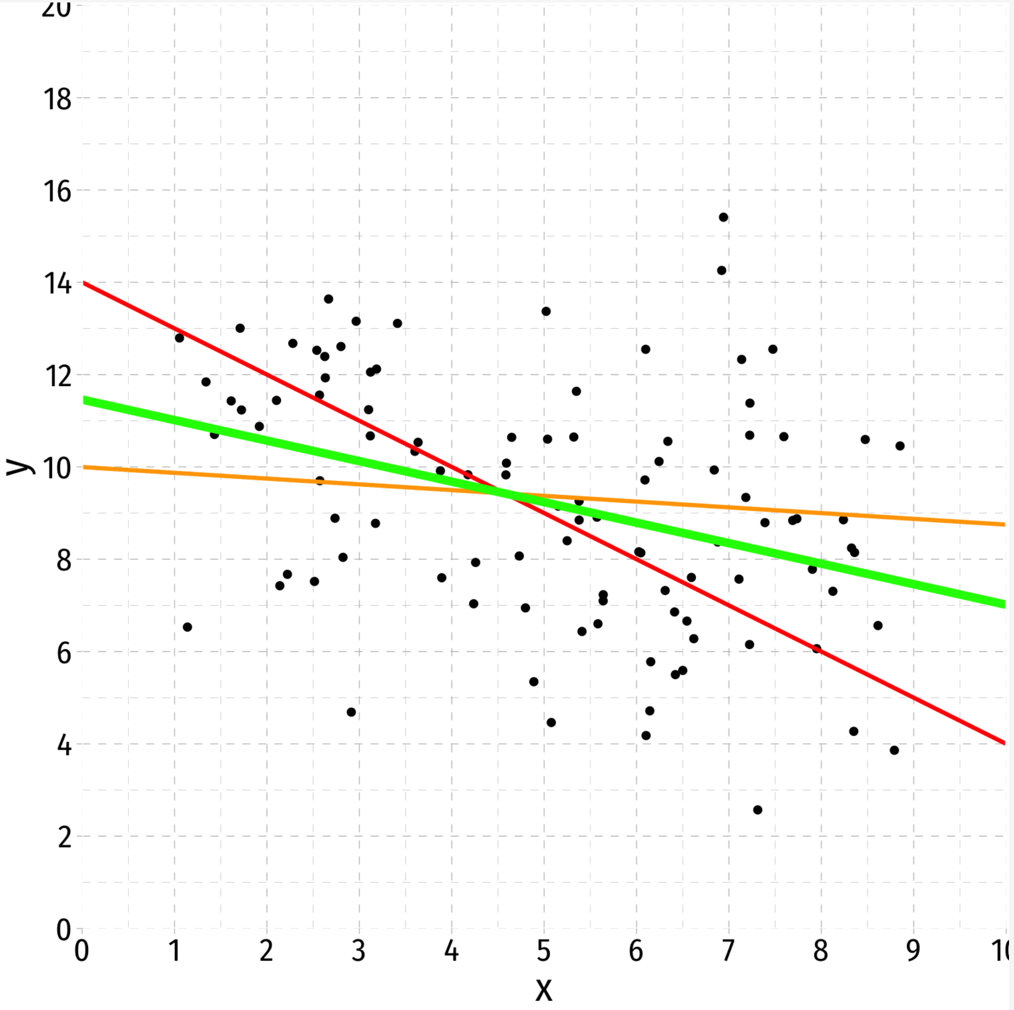


Population Linear Regression Model

- Linear regression lets us estimate the slope of the **population** regression line between X and Y using **sample** data
- We can make **statistical inferences** about the population slope coefficient
  - eventually & hopefully: a ***causal* inference**
- $\text{slope} = \frac{\Delta Y}{\Delta X}$: for a 1-unit change in X, how many units will this *cause* Y to change?

Class Size Example

Example: What is the relationship between class size and educational performance?

Class Size Example: Load the Data

```r
# install.packages("haven") # install for first use
library("haven") # load for importing .dta files
CASchool<-read_dta("../data/caschool.dta")
```

Class Size Example: Look at the Data I

```r
glimpse(CASchool)
```

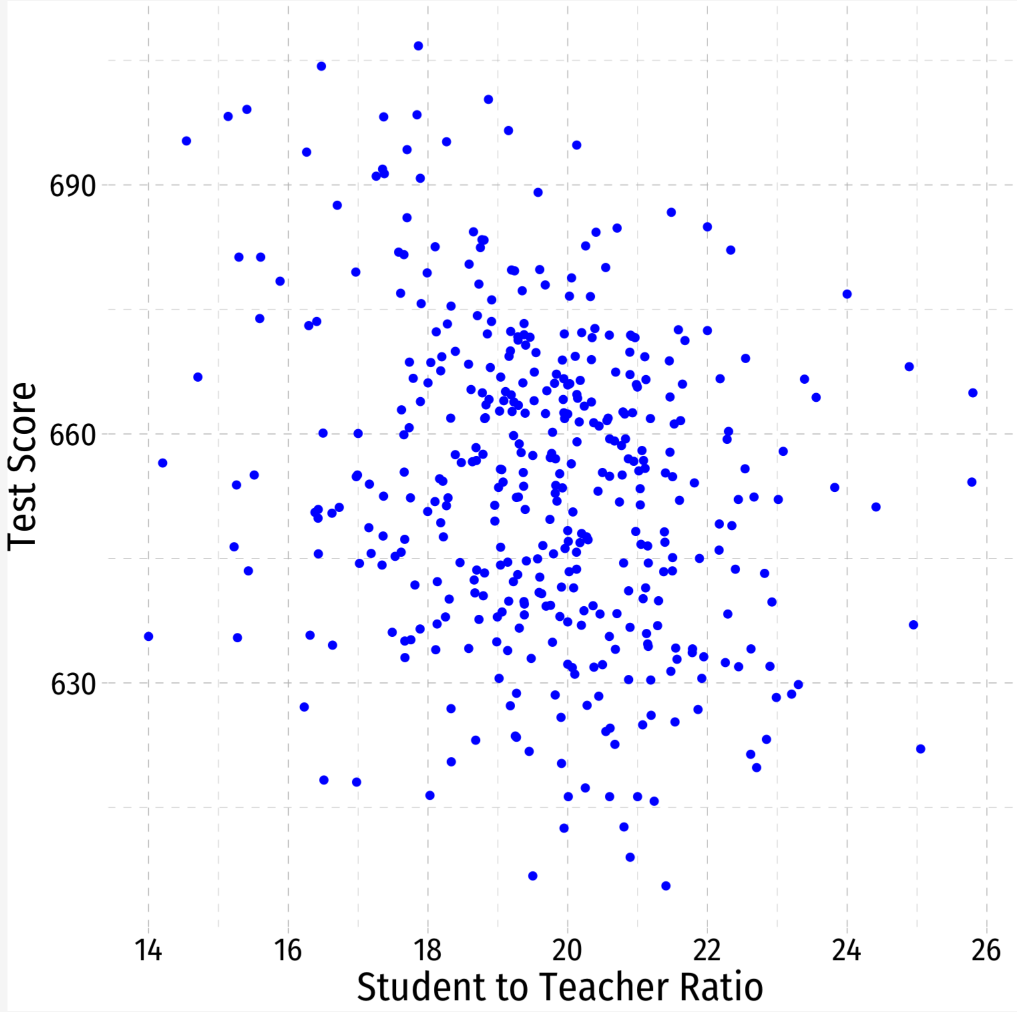
```
## Rows: 420
## Columns: 21
## $ observat <dbl> 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 1…
## $ dist_cod <dbl> 75119, 61499, 61549, 61457, 61523, 62042, 68536, 63834, 6233…
## $ county   <chr> "Alameda", "Butte", "Butte", "Butte", "Butte", "Fresno", "Sa…
## $ district <chr> "Sunol Glen Unified", "Manzanita Elementary", "Thermalito Un…
## $ gr_span  <chr> "KK-08", "KK-08", "KK-08", "KK-08", "KK-08", "KK-08", "KK-08…
## $ enrl_tot <dbl> 195, 240, 1550, 243, 1335, 137, 195, 888, 379, 2247, 446, 98…
## $ teachers <dbl> 10.90, 11.15, 82.90, 14.00, 71.50, 6.40, 10.00, 42.50, 19.00…
## $ calw_pct <dbl> 0.5102, 15.4167, 55.0323, 36.4754, 33.1086, 12.3188, 12.9032…
## $ meal_pct <dbl> 2.0408, 47.9167, 76.3226, 77.0492, 78.4270, 86.9565, 94.6237…
## $ computer <dbl> 67, 101, 169, 85, 171, 25, 28, 66, 35, 0, 86, 56, 25, 0, 31,…
## $ testscr  <dbl> 690.80, 661.20, 643.60, 647.70, 640.85, 605.55, 606.75, 609.…
## $ comp_stu <dbl> 0.34358975, 0.42083332, 0.10903226, 0.34979424, 0.12808989, …
## $ expn_stu <dbl> 6384.911, 5099.381, 5501.955, 7101.831, 5235.988, 5580.147, …
## $ str      <dbl> 17.88991, 21.52466, 18.69723, 17.35714, 18.67133, 21.40625, …
## $ avginc   <dbl> 22.690001, 9.824000, 8.978000, 8.978000, 9.080333, 10.415000…
## $ el_pct   <dbl> 0.000000, 4.583333, 30.000002, 0.000000, 13.857677, 12.40875…
```

Class Size Example: Look at the Data II

observat	dist_cod	county	district	gr_span	enrl_tot	teachers	calw_pct	meal_pct	computer	testscr	comp_stu	expn_stu	str	avginc	el_pct	read_scr	math_scr	aowijef	es_pct	es_frac
1	75119	Alameda	Sunol Glen Unified	KK-08	195	10.90	0.5102	2.0408	67	690.80	0.3435898	6384.911	17.88991	22.690001	0.000000	691.6	690.0	35.77982	1.000000	0.0100000
2	61499	Butte	Manzanita Elementary	KK-08	240	11.15	15.4167	47.9167	101	661.20	0.4208333	5099.381	21.52466	9.824000	4.583334	660.5	661.9	43.04933	3.583334	0.0358333
3	61549	Butte	Thermalito Union Elementary	KK-08	1550	82.90	55.0323	76.3226	169	643.60	0.1090323	5501.955	18.69723	8.978000	30.000002	636.3	650.9	37.39445	29.000002	0.2900000
4	61457	Butte	Golden Feather Union Elementary	KK-08	243	14.00	36.4754	77.0492	85	647.70	0.3497942	7101.831	17.35714	8.978000	0.000000	651.9	643.5	34.71429	1.000000	0.0100000
5	61523	Butte	Palermo Union Elementary	KK-08	1335	71.50	33.1086	78.4270	171	640.85	0.1280899	5235.988	18.67133	9.080333	13.857677	641.8	639.9	37.34266	12.857677	0.1285768
6	62042	Fresno	Burrel Union Elementary	KK-08	137	6.40	12.3188	86.9565	25	605.55	0.1824818	5580.147	21.40625	10.415000	12.408759	605.7	605.4	42.81250	11.408759	0.1140876

Class Size Example: Scatterplot

```r
scatter <- ggplot(data = CASchool)+
  aes(x = str,
      y = testscr)+
  geom_point(color = "blue")+
  labs(x = "Student to Teacher Ratio",
       y = "Test Score")+
  theme_pander(base_family = "Fira Sans Condensed",
           base_size = 20)
scatter
```



Class Size Example: Slope I

- If we *change* (Δ) the class size by an amount, what would we expect the *change* in test scores to be?

$$\beta = \frac{\text{change in test score}}{\text{change in class size}} = \frac{\Delta \text{test score}}{\Delta \text{class size}}$$

- If we knew β, we could say that changing class size by 1 student will change test scores by β

Class Size Example: Slope II

- Rearranging:

$$\Delta\text{test score} = \beta \times \Delta\text{class size}$$

Class Size Example: Slope II

- Rearranging:

$$\Delta\text{test score} = \beta \times \Delta\text{class size}$$

- Suppose $\beta = -0.6$. If we shrank class size by 2 students, our model predicts:

$$\begin{aligned}\Delta\text{test score} &= -2 \times \beta\\ \Delta\text{test score} &= -2 \times -0.6\\ \Delta\text{test score} &= 1.2\end{aligned}$$

Class Size Example: Slope and Average Effect

$$\text{test score} = \beta_0 + \beta_1 \times \text{class size}$$

- The line relating class size and test scores has the above equation
- β_0 is the **vertical-intercept**, test score where class size is 0
- β_1 is the **slope** of the regression line
- This relationship only holds **on average** for all districts in the population, *individual* districts are also affected by other factors

Class Size Example: Marginal Effects

- To get an equation that holds for *each* district, we need to include other factors

$$\text{test score} = \beta_0 + \beta_1 \text{class size} + \text{other factors}$$

- For now, we will ignore these until Unit III

- Thus, $\beta_0 + \beta_1 \text{class size}$ gives the **average effect** of class sizes on scores

- Later, we will want to estimate the **marginal effect** (**causal effect**) of each factor on an individual district's test score, holding all other factors constant

Econometric Models Overview

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + u$$

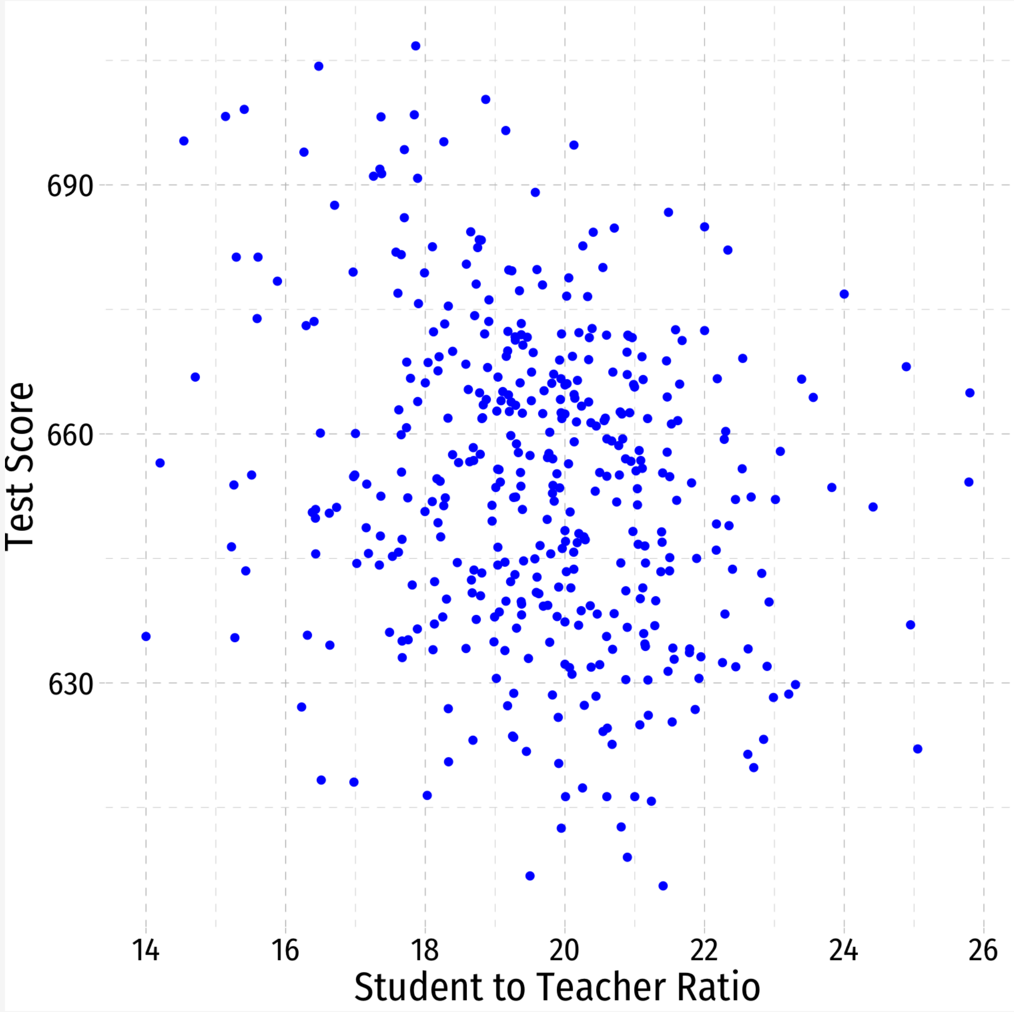
- Y is the **dependent variable** of interest
  - AKA "response variable," "regressand," "Left-hand side (LHS) variable"
- X_1 and X_2 are **independent variables**
  - AKA "explanatory variables", "regressors," "Right-hand side (RHS) variables", "covariates"
- Our data consists of a spreadsheet of observed values of (X_{1i}, X_{2i}, Y_i)
- To model, we **"regress Y on X_1 and X_2"**
- β_0 and β_1 are **parameters** that describe the population relationships between the variables
  - unknown! to be estimated!
- u is the random **error term**
  - **'U'nobservable**, we can't measure it, and must model with assumptions about it

The Population Regression Model

- How do we draw a line through the scatterplot? We do not know the **"true"** β_0 or β_1

- We do have data from a *sample* of class sizes and test scores[†]

- So the real question is, **how can we estimate β_0 and β_1?**

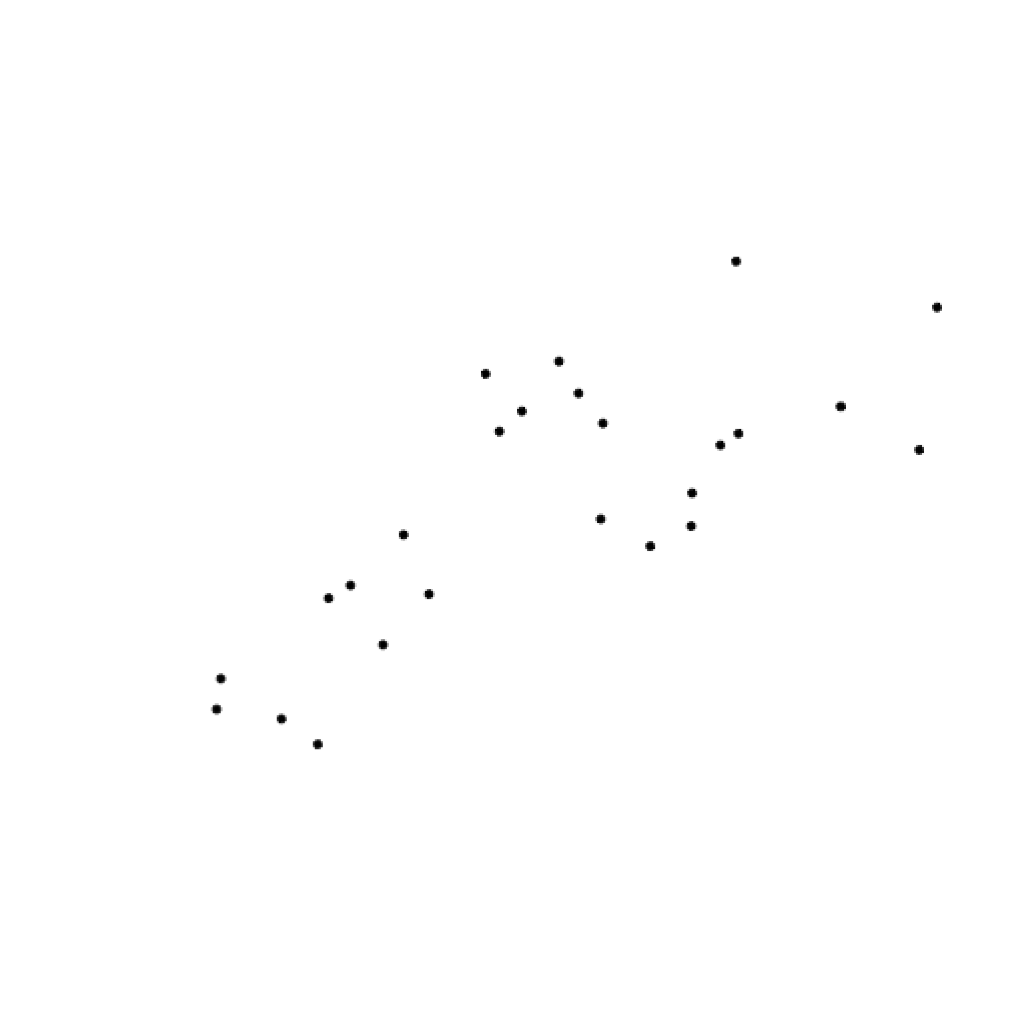


[†] Data are student-teacher-ratio and average test scores on Stanford 9 Achievement Test for 5th grade students for 420 K-6 and K-8 school districts in California in 1999, (Stock and Watson, 2015: p. 141)

Deriving OLS

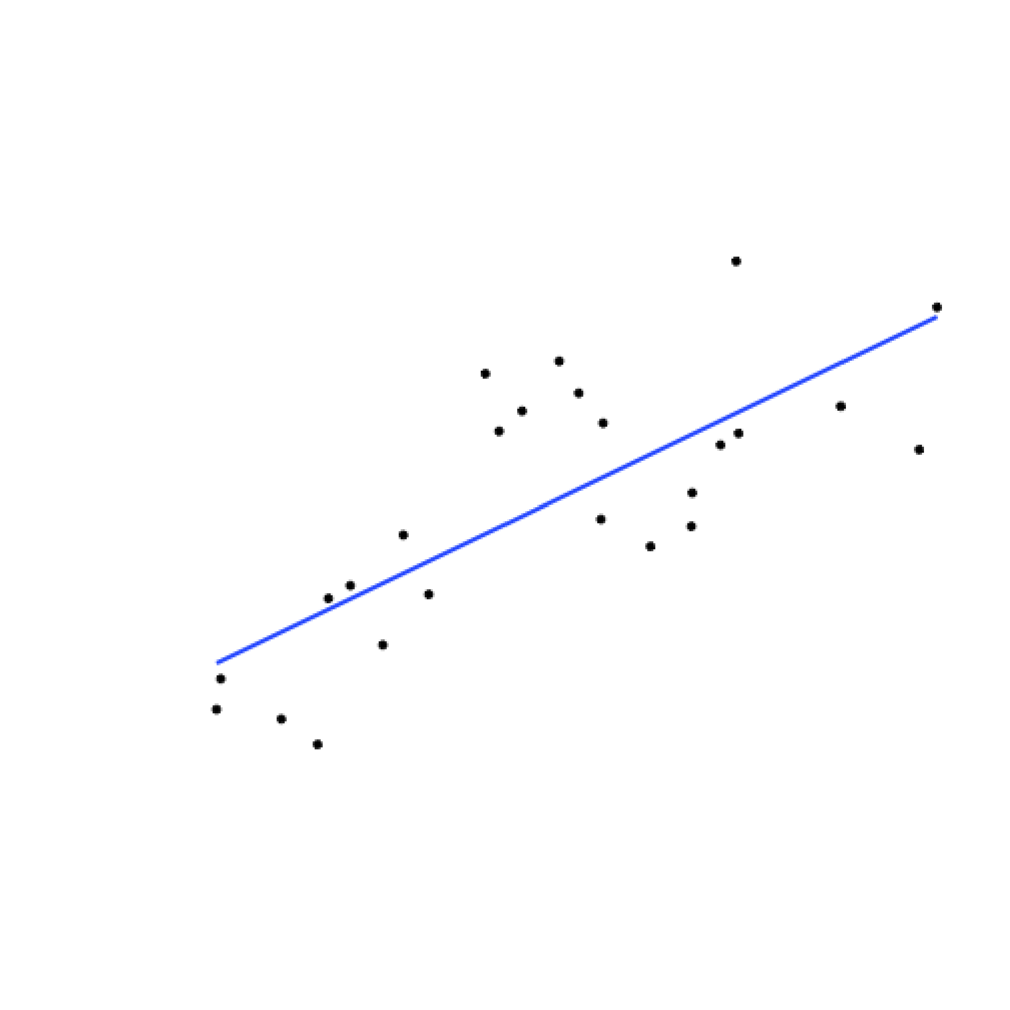
Deriving OLS

- Suppose we have some data points



Deriving OLS

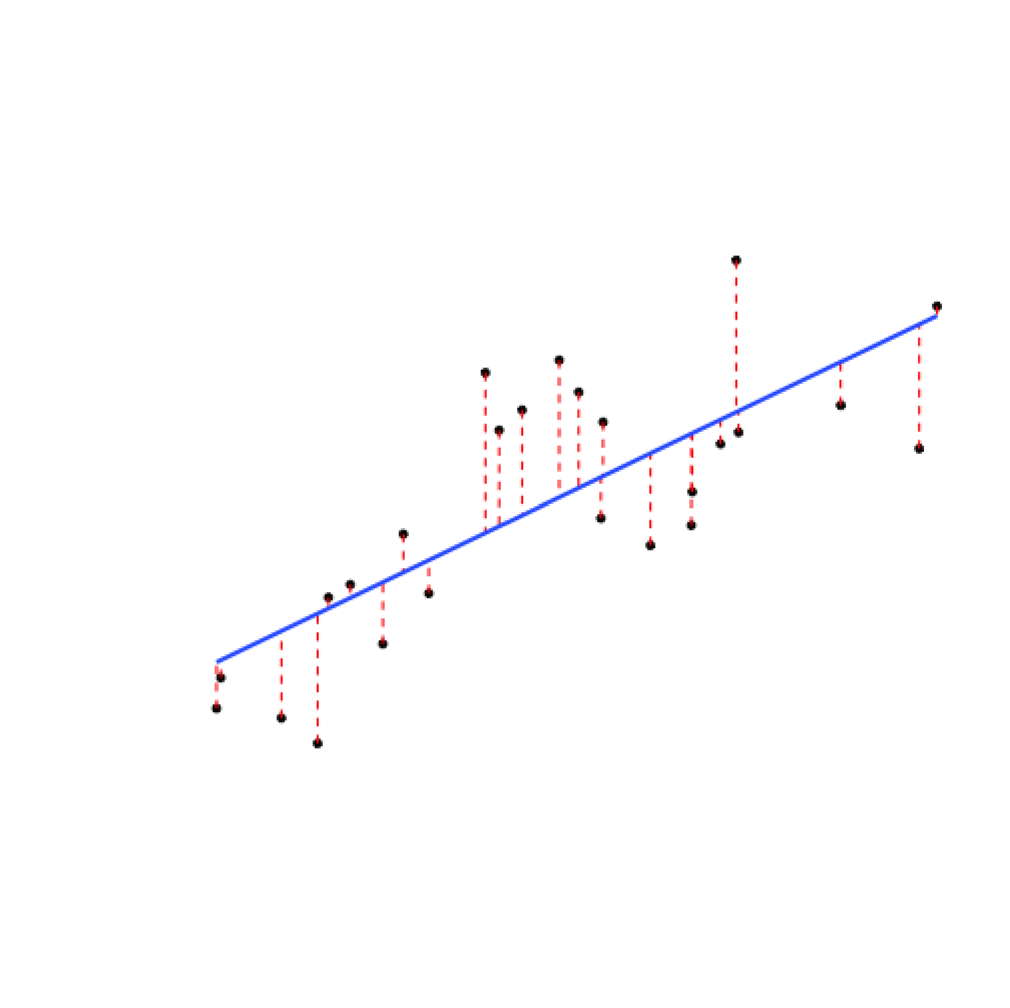
- Suppose we have some data points
- We add a line



Deriving OLS

- Suppose we have some data points
- We add a line
- The **residual**, $\hat{u}$ of each data point is the difference between the **actual** and the **predicted** value of Y given X:

$$u_i = Y_i - \hat{Y}_i$$



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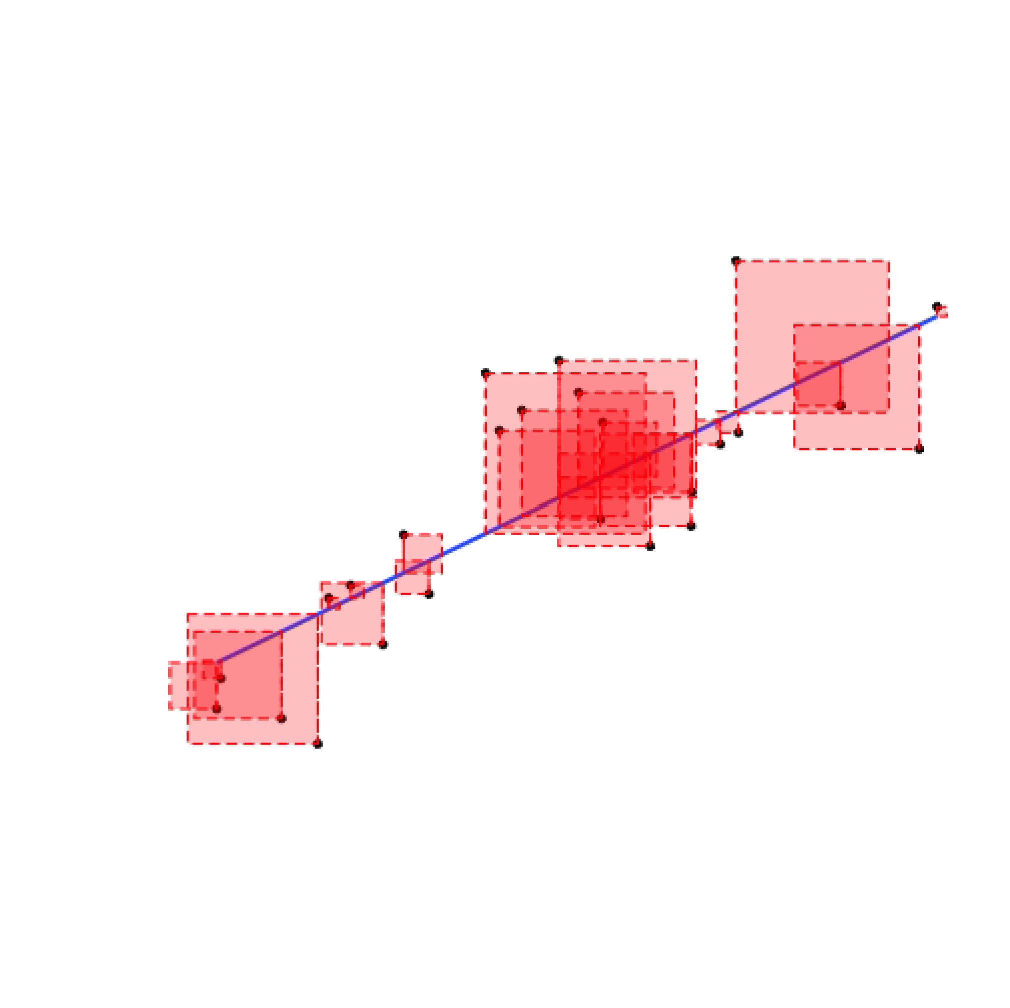

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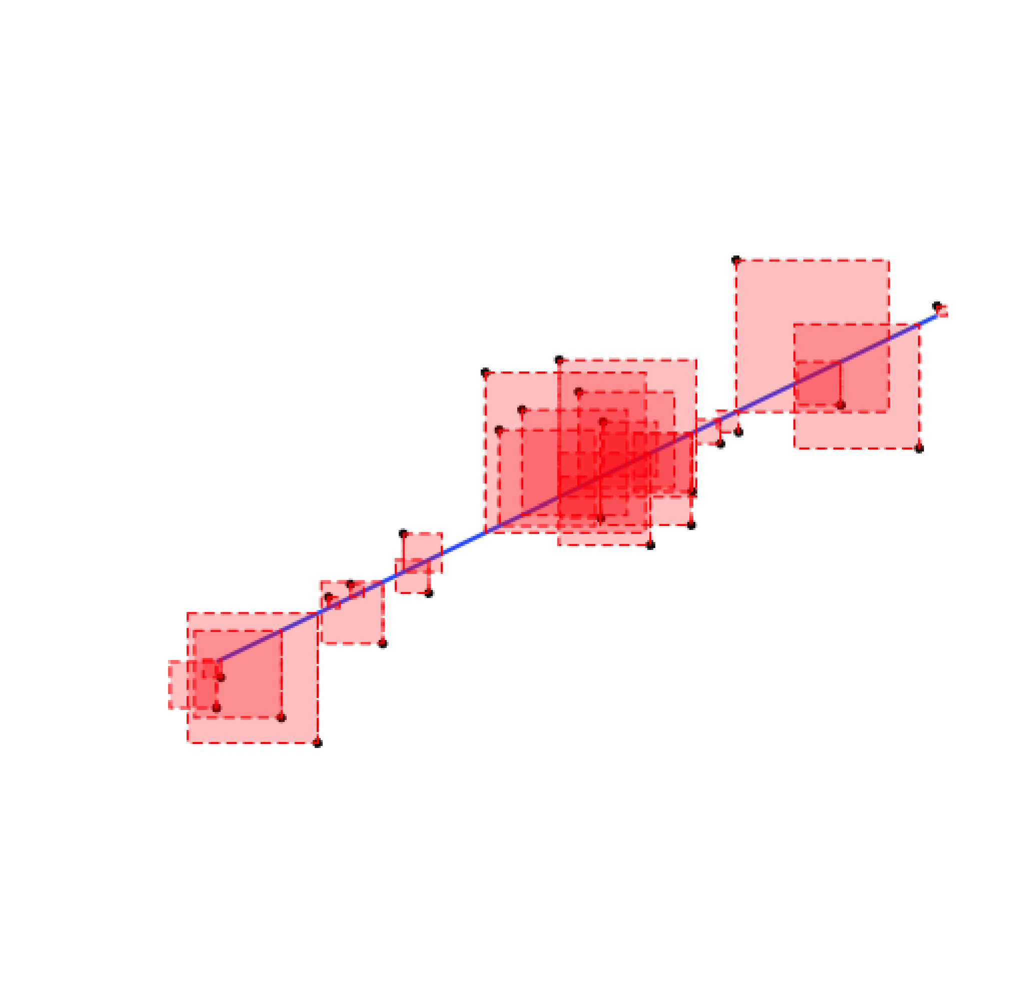
- Add all of these up: **Sum of Squared Errors (SSE)**

$$SSE = \sum_{i=1}^{n} u_i^2$$



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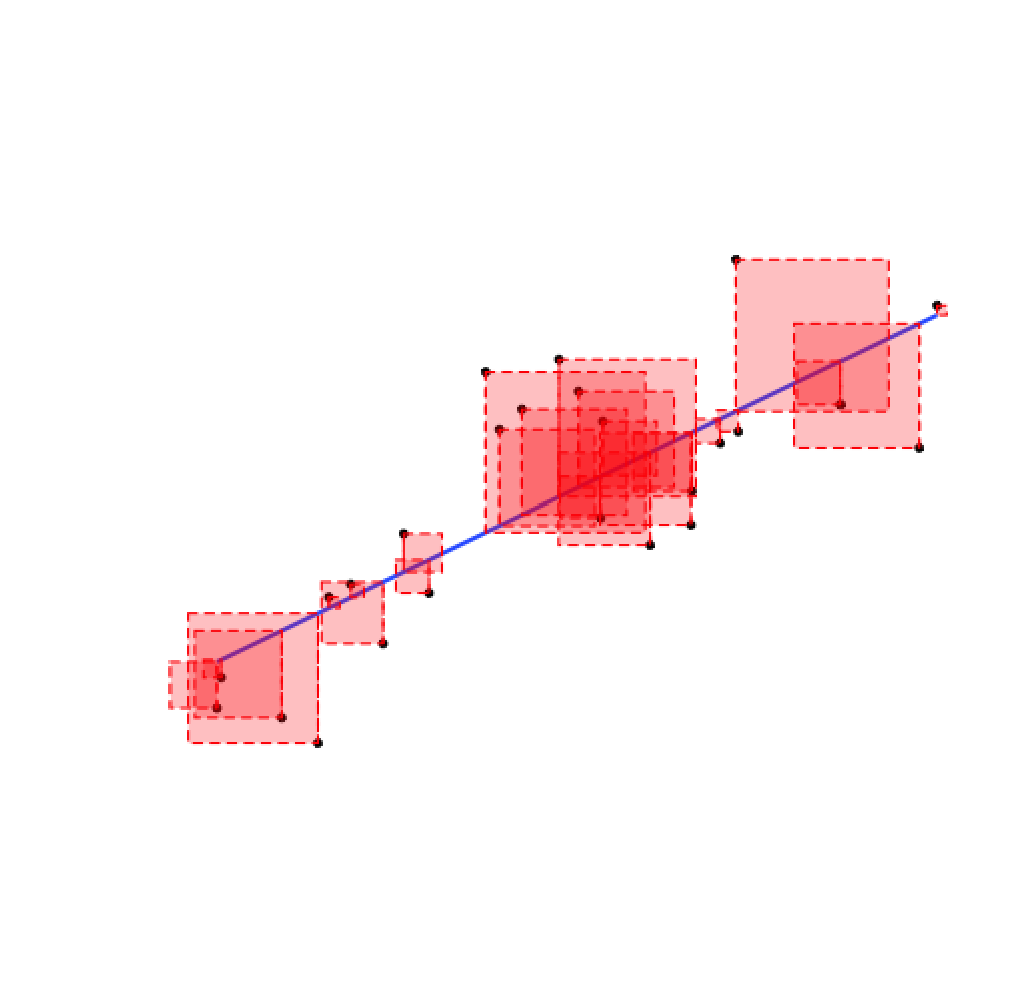
$$SSE = \sum_{i=1}^{n} u_i^2$$

- **The line of best fit *minimizes* SSE**



*O*rdinary *L*east *S*quares Estimators

- The **Ordinary Least Squares (OLS) estimators** of the unknown population parameters β_0 and β_1, solve the calculus problem:

$$\min_{\beta_0, \beta_1} \sum_{i=1}^{n} \underbrace{[Y_i - (\underbrace{\beta_0 + \beta_1 X_i}_{\hat{Y}_i})]}_{u}^2$$

- Intuitively, OLS estimators **minimize the average squared distance between the actual values (Y_i) and the predicted values $(\hat{Y}_i)$ along the estimated regression line**

The OLS Regression Line

- The **OLS regression line** or **sample regression line** is the linear function constructed using the OLS estimators:

$$\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i$$

- $\hat{\beta}_0$ and $\hat{\beta}_1$ ("beta 0 hat" & "beta 1 hat") are the **OLS estimators** of population parameters β_0 and β_1 using sample data

- The **predicted value** of Y given X, based on the regression, is $E(Y_i|X_i) = \hat{Y}_i$

- The **residual** or **prediction error** for the i^{th} observation is the difference between observed Y_i and its predicted value, $\hat{u}_i = Y_i - \hat{Y}_i$

The OLS Regression Estimators

- The solution to the SSE minimization problem yields:[†]

$$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X}$$

$$\hat{\beta}_1 = \frac{\displaystyle\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y})}{\displaystyle\sum_{i=1}^{n}(X_i - \bar{X})^2} = \frac{s_{XY}}{s_X^2} = \frac{cov(X, Y)}{var(X)}$$

[†] See next's class notes page for proofs.

Our Class Size Example in R

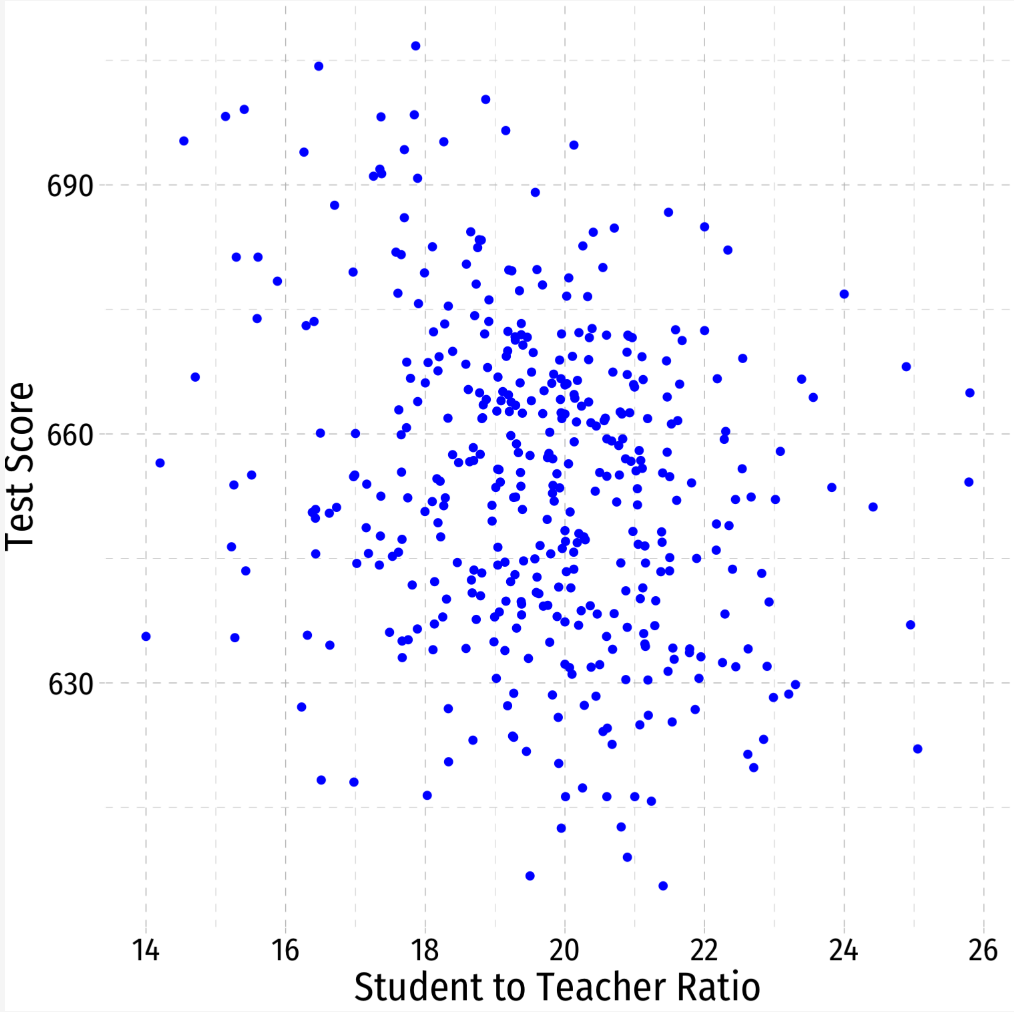
Class Size Scatterplot (Again)

```
scatter
```

- There is some true (unknown) population relationship:

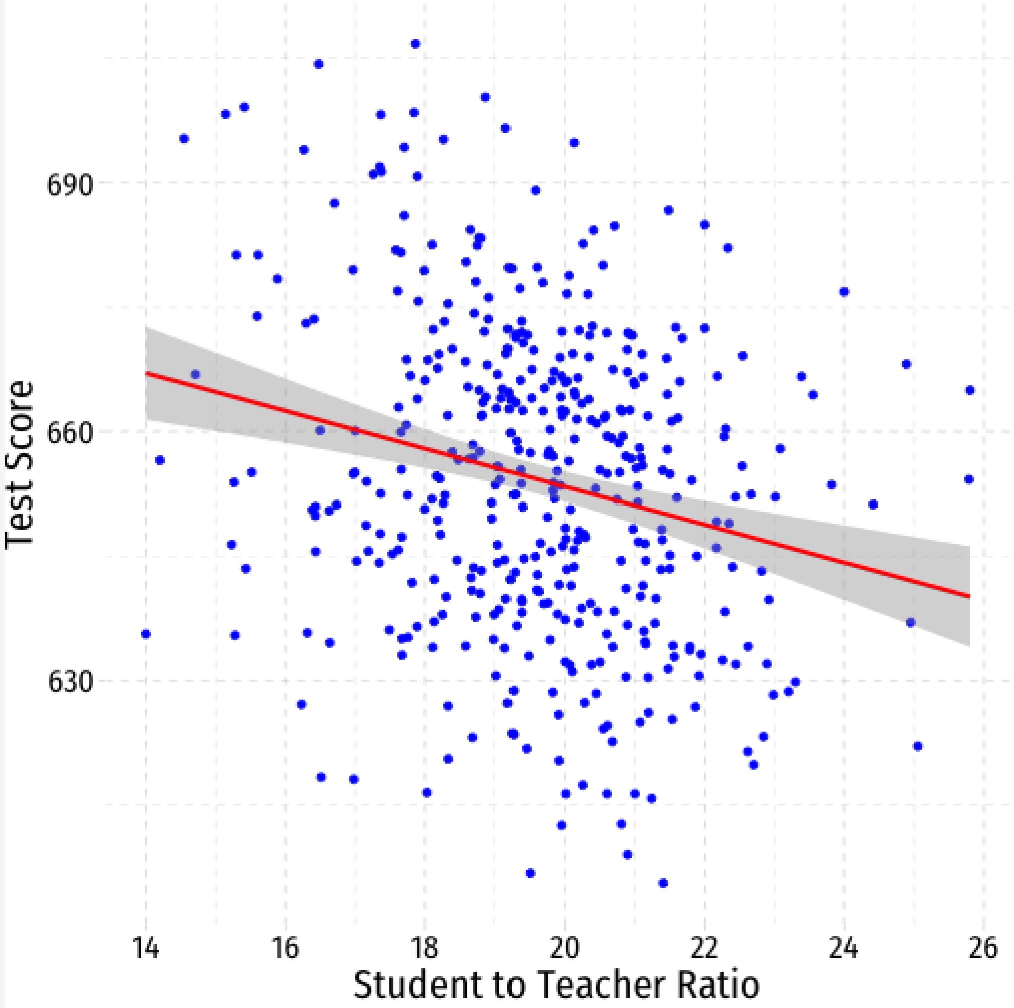
$$\text{test score} = \beta_0 + \beta_1 \times str$$

- $\beta_1 = \frac{\Delta \text{test score}}{\Delta \text{str}} = ??$



Class SIze Scatterplot with Regression Line

```r
scatter+
  geom_smooth(method = "lm", color = "red")
```



OLS in R

```r
# run regression of testscr on str
school_reg <- lm(testscr ~ str,
                 data = CASchool)
```

Format for regression is `lm(y ~ x, data = df)`

- `y` is dependent variable (listed first!)
- `~` means "modeled by" or "explained by"
- `x` is the independent variable
- `df` is name of dataframe where data is stored

OLS in R II

```r
# look at reg object
school_reg
```

```
##
## Call:
## lm(formula = testscr ~ str, data = CASchool)
##
## Coefficients:
## (Intercept)          str
##      698.93        -2.28
```

- Stored as an `lm` object called `school_reg`, a `list` object

OLS in R III

- Looking at the `summary`, there's a lot of information here!
- These objects are cumbersome, come from a much older, pre-`tidyverse` epoch of `base R`
- Luckily, we now have `tidy` ways of working with regressions!

```
summary(school_reg) # get full summary
```

```
##
## Call:
## lm(formula = testscr ~ str, data = CASchool)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -47.727 -14.251   0.483  12.822  48.540
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 698.9330     9.4675  73.825  < 2e-16 ***
## str          -2.2798     0.4798  -4.751 2.78e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 18.58 on 418 degrees of freedom
## Multiple R-squared:  0.05124,    Adjusted R-squared:  0.04897
## F-statistic: 22.58 on 1 and 418 DF,  p-value: 2.783e-06
```

Tidy OLS in R: broom I

- The `broom` package allows us to *tidy* up regression objects†
- The `tidy()` function creates a *tidy* `tibble` of regression output

```
# load packages
library(broom)

# tidy regression output
tidy(school_reg)
```

```
## # A tibble: 2 x 5
##   term        estimate std.error statistic   p.value
##   <chr>          <dbl>     <dbl>     <dbl>     <dbl>
## 1 (Intercept)   699.       9.47      73.8  6.57e-242
## 2 str            -2.28     0.480     -4.75 2.78e-  6
```

† See more at broom.tidyverse.org.

Tidy OLS in R: broom II

- The `broom` package allows us to *tidy* up regression objects[†]

- The `tidy()` function creates a *tidy* `tibble` of regression output

```
# load packages
library(broom)

# tidy regression output (with confidence intervals!)
tidy(school_reg,
     conf.int = TRUE)
```

```
## # A tibble: 2 x 7
##   term        estimate std.error statistic   p.value conf.low conf.high
##   <chr>          <dbl>     <dbl>     <dbl>     <dbl>    <dbl>     <dbl>
## 1 (Intercept)   699.       9.47      73.8  6.57e-242   680.      718.
## 2 str            -2.28     0.480     -4.75 2.78e-  6    -3.22     -1.34
```

† See more at broom.tidyverse.org.

More broom Tools: glance

- `glance()` shows us a lot of overall regression statistics and diagnostics
  - We'll interpret these in the next lecture and beyond

```
# look at regression statistics and diagnostics
glance(school_reg)
```

```
## # A tibble: 1 x 12
##   r.squared adj.r.squared sigma statistic p.value    df logLik   AIC   BIC
##       <dbl>         <dbl> <dbl>     <dbl>   <dbl> <dbl>  <dbl> <dbl> <dbl>
## 1    0.0512        0.0490  18.6      22.6 2.78e-6     1 -1822. 3650. 3663.
## # … with 3 more variables: deviance <dbl>, df.residual <int>, nobs <int>
```

More broom Tools: augment

- `augment()` creates useful new variables in the stored `lm` object
  - `.fitted` are fitted (predicted) values from model, i.e. $\hat{Y}_i$
  - `.resid` are residuals (errors) from model, i.e. $\hat{u}_i$

```r
# add regression-based values to data
augment(school_reg)
```

```
## # A tibble: 420 x 8
##    testscr   str .fitted .resid .std.resid    .hat .sigma  .cooksd
##      <dbl> <dbl>   <dbl>  <dbl>      <dbl>   <dbl>  <dbl>    <dbl>
##  1    691.  17.9    658.   32.7      1.76  0.00442   18.5 0.00689
##  2    661.  21.5    650.   11.3      0.612 0.00475   18.6 0.000893
##  3    644.  18.7    656.  -12.7     -0.685 0.00297   18.6 0.000700
##  4    648.  17.4    659.  -11.7     -0.629 0.00586   18.6 0.00117
##  5    641.  18.7    656.  -15.5     -0.836 0.00301   18.6 0.00105
##  6    606.  21.4    650.  -44.6     -2.40  0.00446   18.5 0.0130
##  7    607.  19.5    654.  -47.7     -2.57  0.00239   18.5 0.00794
##  8    609   20.9    651.  -42.3     -2.28  0.00343   18.5 0.00895
##  9    612.  19.9    653.  -41.0     -2.21  0.00244   18.5 0.00597
## 10    613.  20.8    652.  -38.9     -2.09  0.00329   18.5 0.00723
## # … with 410 more rows
```

Class Size Regression Result I

- Using OLS, we find:

$$\widehat{\text{test score}} = 689.9 - 2.28 \times str$$

Class Size Regression Result II

- There's a great package called `equatiomatic` that prints this equation in `markdown` or $\LaTeX$.

$$\operatorname{testscr} = 698.93 - 2.28(\operatorname{str}) + \epsilon$$

Here was my code:

```r
# install.packages("equatiomatic") # install for first use
library(equatiomatic) # load it
extract_eq(school_reg, # regression lm object
           use_coefs = TRUE, # use names of variables
           coef_digits = 2, # round to 2 digits
           fix_signs = TRUE) # fix negatives (instead of + -)
```

```
## $$
## \operatorname{testscr} = 698.93 - 2.28(\operatorname{str}) + \epsilon
## $$
```

Class Size Regression: A Data Point

- One district in our sample is Richmond, CA:

```
CASchool %>%
  filter(district=="Richmond Elementary") %>%
  dplyr::select(district, testscr, str)
```

```
## # A tibble: 1 x 3
##   district            testscr   str
##   <chr>                 <dbl> <dbl>
## 1 Richmond Elementary    672.    22
```

- Predicted value:

$$\widehat{\text{Test Score}}_{\text{Richmond}} = 698 - 2.28(22) \approx 648$$

- Residual

$$\hat{u}_{Richmond} = 672 - 648 \approx 24$$