4.2 — Difference-in-Difference Models ECON 480 • Econometrics • Fall 2020 Ryan Safner Assistant Professor of Economics ✓ safner@hood.edu ○ ryansafner/metricsF20 ⓒ metricsF20.classes.ryansafner.com

Outline

Difference-in-Difference Models

- Example I: HOPE in Georgia
- **Generalizing DND Models**
- Example II: "The" Card-Kreuger Minimum Wage Study

Clever Research Designs Identify Causality



Again, **this toolkit** of research designs to **identify causal effects** is the economist's **comparative advantage** that firms and governments want!



Natural Experiments



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• The difference-in-difference model (aka "diff-in-diff" or "DND") identifies treatment effect by differencing the difference pre- and post-treatment values of \(Y\) between treatment and control groups

\$\$\hat{Y_{it}}=\beta_0+\beta_1 \text{Treated}_i +\beta_2 \text{After}_{t}+\beta_3 (\text{Treated}_i \times
\text{After}_{t})+u_{it}\$\$

\(Treated_i= \begin{cases}1 \text{ if } i \text{ is in treatment group}\\ 0 \text{ if } i \text{ is not in treatment group}\end{cases} \quad After_t= \begin{cases}1 \text{ if } t \text{ is after treatment period}\\ 0 \text{ if } t \text{ is before treatment period}\\ 0 \text{ is before treatment period}\\ 0 \text{ if } t \text{ is before treatment period}\\ 0 \text{ if } t \text{ is before treatment period}\\ 0 \text{ if } t \text{ is before treatment period}\\ 0 \text{ if } t \text{ is before treatment period}\\ 0 \text{ if } t \text{ is before treatment period}\\ 0 \text{ if } t \text{ is treatment period}\\ 0 \text{ if } t \text{ is treatment period}\\ 0 \text{ if } t \text{ is treatment period}\\ 0 \text{ if } t \text{ is treatment period}\\ 0 \text{ if } t \text{ is treatment period}\\ 0 \text{ if } t \text{ is treatment period}\\ 0 \text{ if } t \text{ is treatment period}\\ 0 \text{ if } t \text{ is treatment period}\\ 0 \text{ if } t \text{ is treatment period}\\ 0 \text{ is treatment period}\\ 0 \text{

	Control	Treatment	Group Diff \((\Delta Y_i)\)
Before	\(\beta_0\)	\(\beta_0+\beta_1\)	\(\beta_1\)
After	\ (\beta_0+\beta_2\)	\ (\beta_0+\beta_1+\beta_2+\beta_3\)	\(\beta_1+\beta_3\)
Time Diff \((\Delta Y_t)\)	\(\beta_2\)	\(\beta_2+\beta_3\)	Diff-in-diff \(\Delta_i \Delta_t: \beta_3\)

Silly Example: Hot Dogs



Is there a discount when you get cheese and chili?

price	cheese	chili
<dbl></dbl>	<dpl></dpl>	<dpl></dpl>
2.00	0	0
2.35	1	0
2.35	0	1
2.70	1	1
4 rows		

```
lm(price ~ cheese + chili + cheese*chili,
    data = hotdogs) %>%
    tidy()
```

term	estimate
<chr></chr>	<dpl></dpl>
(Intercept)	2.00
cheese	0.35
chili	0.35
cheese:chili	0.00
4 rows	



Silly Example: Hot Dogs



Is there a discount when you get cheese *and* chili?

	No Cheese	Cheese	Cheese Diff
No Chili	\$2.00	\$2.35	\$0.35
Chili	\$2.35	\$2.70	\$0.35
Chili Diff	\$0.35	\$0.35	\$0.00 (Diff-in-diff)

• Diff-n-diff is just a model with an interaction term between two dummies!

```
lm(price ~ cheese + chili + cheese*chili,
    data = hotdogs) %>%
    tidy()
```

term	estimate
<chr></chr>	<dpl></dpl>
(Intercept)	2.00
cheese	0.35
chili	0.35
cheese:chili	0.00
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- Control group \((Treated = 0)\)
- \(\hat{\beta_0}\): value of \(Y\) for control group before treatment
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\$\$\hat{Y_{it}}=\beta_0+\beta_1 \text{Treated}_i +\beta_2 \text{After}_{t}+\beta_3 (\text{Treated}_i \times \text{After}_{t})+u_{it}\$\$



\(\bar{Y_i}\) for Control group before: \
 (\hat{\beta_0}\)





- \(\bar{Y_i}\) for Control group before: \
 (\hat{\beta_0}\)
- \(\bar{Y_i}\) for Control group after: \
 (\hat{\beta_0}+\hat{\beta_2}\)





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 (\hat{\beta_0}+\hat{\beta_2}\)
- \(\bar{Y_i}\) for Treatment group before: \
 (\hat{\beta_0}+\hat{\beta_1}\)





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 (\hat{\beta_0}\)
- \(\bar{Y_i}\) for Control group after: \
 (\hat{\beta_0}+\hat{\beta_2}\)
- \(\bar{Y_i}\) for Treatment group before: \
 (\hat{\beta_0}+\hat{\beta_1}\)
- \(\bar{Y_i}\) for Treatment group after: \
 (\hat{\beta_0}+\hat{\beta_1}+\hat{\beta_2}+\hat{\beta





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 (\hat{\beta_0}\)
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- Group Difference (before): \(\hat{\beta_1}\)





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Comparing Group Means (Again)



	Control	Treatment	Group Diff \ ((\Delta Y_i)\)
Before	\(\beta_0\)	\(\beta_0+\beta_1\)	\(\beta_1\)
After	\ (\beta_0+\beta_2\)	\ (\beta_0+\beta_1+\beta_2+\beta_3\)	\ (\beta_1+\beta_3\)
Time Diff \ ((\Delta Y_t)\)	\(\beta_2\)	\(\beta_2+\beta_3\)	Diff-in-diff \ (\Delta_i \Delta_t: \beta_3\)

Key Assumption: Counterfactual





- Key assumption for DND: time trends (for treatment and control) are parallel
- Treatment and control groups assumed to be identical over time on average, except for treatment
- Counterfactual: if the treatment group had not recieved treatment, it would have changed identically over time as the control group \((\hat{\beta_2})\)

Key Assumption: Counterfactual



\$\$\hat{Y_{it}}=\beta_0+\beta_1 \text{Treated}_i +\beta_2 \text{After}_{t}+\beta_3
(\text{Treated}_i \times \text{After}_{t})+u_{it}\$\$



 If the time-trends would have been *different*, a **biased** measure of the treatment effect \((\hat{\beta_3})\)!



Example I: HOPE in Georgia

Diff-in-Diff Example I

Example: In 1993 Georgia initiated a HOPE scholarship program to let state residents with at least a B average in high school attend public college in Georgia for free. Did it increase college enrollment?

- Micro-level data on 4,291 young individuals
- \(\text{InCollege}_{it}=\begin{cases}1 \text{ if } i \text{ is in college during year }t\\ 0 \text{ if } i \text{ is not in college during year }t\\ \end{cases}\)
- \(\text{Georgia}_i=\begin{cases}1 \text{ if } i \text{ is a Georgia resident}\\ 0 \text{ if } i \text{ is not a Georgia resident}\\ \end{cases}\)
- \(\text{After}_t=\begin{cases}1 \text{ if } t \text{ is after 1992}\\ 0 \text{ if } t \text{ is after 1992}\\ \end{cases}\)

Dynarski, Susan (2000), "Hope for Whom? Financial Aid for the Middle Class and Its Impact on College Attendance"

Diff-in-Diff Example II

- We can use a DND model to measure the effect of HOPE scholarship on enrollments
- Georgia and nearby States, if not for HOPE, changes should be the same over time
- Treatment period: after 1992
- Treatment: Georgia
- Differences-in-differences: \$\$\Delta_i \Delta_t Enrolled = (\text{GA}_{after}-\text{GA}_{before})-(\text{neighbors}_{after}-\text{neighbors}_{before})\$\$
- Regression equation: \$\$\widehat{Enrolled_{it}} = \beta_0+\beta_1 \, Georgia_{i}+\beta_2 \, After_{t}+\beta_3 \, (Georgia_{i} \times After_{t})\$\$

Example: Data

StateCode	Age Year	Weight	Age18	LowIncome	InCollege	After	Georgia	AfterGeorgia
<fctr></fctr>								<dbl></dbl>
56	19 89	1396	0	1	1	0	0	0
56	19 89	1080	0	NA	1	0	0	0
56	18 89	924	1	1	1	0	0	0
56	19 89	891	0	0	1	0	0	0
56	19 89	1395	0	NA	0	0	0	0
56	18 89	1106	1	1	1	0	0	0
56	19 89	965	0	NA	0	0	0	0
56	18 89	958	1	NA	0	0	0	0
56	19 89	1006	0	NA	0	0	0	0
56	19 89	1183	0	1	1	0	0	0
1-10 of 4,291 r	ows 1-10 of	11 column	S			Previou	s 1 2 3	4 5 6 430Next

Example: Data





Example: Regression



DND_reg<-lm(InCollege ~ Georgia + After + Georgia*After, data = hope)
DND_reg %>% tidy()

term	estimate	std.error	statistic	p.value
<chr></chr>				<dbl></dbl>
(Intercept)	0.405782652	0.01092390	37.1463182	4.221545e-262
Georgia	-0.105236204	0.03778114	-2.7854165	5.369384e-03
After	-0.004459609	0.01585224	-0.2813235	7.784758e-01
Georgia:After	0.089329828	0.04889329	1.8270364	6.776378e-02
4 rows				

\$\$\widehat{Enrolled_{it}}=0.406-0.105 \, Georgia_{i}-0.004 \, After_{t}+0.089 \, (Georgia_{i} \times After_{t})\$\$

Example: Interpretting the Regression



\$\$\widehat{Enrolled_{it}}=0.406-0.105 \, Georgia_{i}-0.004 \, After_{t}+0.089 \, (Georgia_{i} \times After_{t})\$

- \(\beta_0\): A **non-Georgian before** 1992 was 40.6% likely to be a college student
- \(\beta_1\): Georgians before 1992 were 10.5% less likely to be college students than neighboring states
- \(\beta_2\): **After** 1992, **non-Georgians** are 0.4% less likely to be college students
- \(\beta_3\): **After** 1992, **Georgians** are 8.9% more likely to enroll in colleges than neighboring states
- Treatment effect: HOPE increased enrollment likelihood by 8.9%

Example: Comparing Group Means



\$\$\widehat{Enrolled_{it}}=0.406-0.105 \, Georgia_{i}-0.004 \, After_{t}+0.089 \, (Georgia_{i} \times After_{t})\$

- A group mean for a dummy \(Y\) is \(E[Y=1]\), i.e. the probability a student is enrolled:
- Non-Georgian enrollment probability pre-1992: \(\beta_0=0.406\)
- Georgian enrollment probability pre-1992: \(\beta_0+\beta_1=0.406-0.105=0.301\)
- Non-Georgian enrollment probability post-1992: \(\beta_0+\beta_2=0.406-0.004=0.402\)
- Georgian enrollment probability post-1992: \(\beta_0+\beta_1+\beta_2+\beta_3=0.406-0.105-0.004+0.089=0.386\)

Example: Comparing Group Means in R





	prob
	<qpf></qpf>
	0.4057827
row	

Example: Comparing Group Means in R II





	prob
	<dbl></dbl>
	0.3005464
row	

Example: Diff-in-Diff Summary



 $\scriptstyle fi=0.406-0.105 \, Georgia_{i}-0.004 \, After_{t}+0.089 \, (Georgia_{i}-10004 \, After_{t})$

	Neighbors	Georgia	Group Diff \((\Delta Y_i)\)
Before	\(0.406\)	\(0.301\)	\(-0.105\)
After	\(0.402\)	\(0.386\)	\(0.016\)
Time Diff \((\Delta Y_t)\)	\(-0.004\)	\(0.085\)	Diff-in-diff : \(0.089\)

\$\$\begin{align*} \Delta_i \Delta_t Enrolled &= (\text{GA}_{after}-\text{GA}_{before})-(\text{neighbors}_{after}-\text{neighbors}_{before})\\ &=(0.386-0.301)-(0.402-0.406)\\ &= (0.085)-(-0.004)\\ &=0.089\\ \end{align*}\$\$

Example: Diff-in-Diff Graph





Generalizing DND Models

Generalizing DND Models

- DND can be generalized with a two-way fixed effects model: \$\$\widehat{Y_{it}}=\alpha_i+\theta_t+\beta_3 (\text{Treated}_i * \text{After}_{t})+\nu_{it}\$\$
 - \(\alpha_i\): group fixed effects (treatments/control groups)
 - o \(\theta_t\): time fixed effects (pre/post treatment)
- Allows *many* periods, and treatment(s) can occur at different times to different units (so long as some do not get treated)
- Can also add control variables that vary within units and over time \$\$\widehat{Y_{it}}=\alpha_i+\theta_t+\beta_3 \, (\text{Treated}_i \times \text{After}_{t})+\beta_4 X_{it}+\nu_{it}\$\$

Our Example, Generalized I



\$\$\widehat{Enrolled_{it}} = \alpha_i+\theta_t+\beta_3 \, (Georgia_{it} \times After_{it})\$\$

- **StateCode** is a variable for the State \(\implies\) create State fixed effect
- Year is a variable for the year \(\implies\) create year fixed effect

Our Example, Generalized II



• Using LSDV method...

term	estimate	std.error	statistic	p.value
<chr></chr>				<dbl></dbl>
(Intercept)	0.418057478	0.02261133	18.4888517	1.734550e-73
Georgia	-0.141501255	0.03936119	-3.5949436	3.281224e-04
After	0.075340594	0.03128021	2.4085706	1.605717e-02
factor(StateCode)57	-0.014181112	0.02739708	-0.5176140	6.047544e-01
factor(StateCode)58	NA	NA	NA	NA
factor(StateCode)59	-0.062378540	0.01954266	-3.1919172	1.423556e-03
factor(StateCode)62	-0.132650271	0.02806143	-4.7271383	2.350298e-06
factor(StateCode)63	-0.005103868	0.02627780	-0.1942274	8.460071e-01
factor(Year)90	0.046608845	0.02833625	1.6448486	1.000745e-01
factor(Year)91	0.032275789	0.02856877	1.1297577	2.586417e-01
1-10 of 17 rows				Previous 1 2 Next

\$\$\widehat{InCollege_{it}}=\alpha_i+\theta_t+0.091 \, (\text{Georgia}_i \times \text{After}_{it})\$\$

Intuition behind DND

- Diff-in-diff models are the quintessential example of exploiting **natural experiments**
- A major change at a point in time (change in law, a natural disaster, political crisis) separates groups where one is affected and another is not---identifies the effect of the change (treatment)
- One of the cleanest and clearest causal **identification strategies**



Example II: "The" Card-Kreuger Minimum Wage Study

Example: "The" Card-Kreuger Minimum Wage Study I



Example: *The* controversial minimum wage study, Card & Kreuger (1994) is a quintessential (and clever) diff-in-diff.

Card, David, Krueger, Alan B, (1994), "Minimum Wages and Employment: A Case Study of the Fast-Food Industry in New Jersey and Pennsylvania," *American Economic Review* 84 (4): 772–793

Card & Kreuger (1994): Background I

- Card & Kreuger (1994) compare employment in fast food restaurants on New Jersey and Pennsylvania sides of border between February and November 1992.
- Pennsylvania & New Jersey both had a minimum wage of \$4.25 before February 1992
- In February 1992, New Jersey raised minimum wage from \$4.25 to \$5.05



Card & Kreuger (1994): Background II

- If we look only at New Jersey before & after change:
 - Omitted variable bias:
 - macroeconomic variables (there's a recession!), weather, etc.
 - Including PA as a control will control for these time-varying effects if they are national trends
- Surveyed 400 fast food restaurants on each side of the border, before & after min wage increase



Card & Kreuger (1994): Comparisons





Card & Kreuger (1994): Summary I



TABLE 1-SAMPLE DE	SIGN AND	Response	RATES
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	All	Stores in:	
		NJ	PA
Wave 1, February 15–March 4, 1992:			
Number of stores in sample frame: ^a	473	364	109
Number of refusals:	63	33	30
Number interviewed:	410	331	79
Response rate (percentage):	86.7	90.9	72.5
Wave 2, November 5 – December 31, 1992:			
Number of stores in sample frame:	410	331	79
Number closed:	6	5	1
Number under rennovation:	2	2	0
Number temporarily closed: ^b	2	2	0
Number of refusals:	1	1	0
Number interviewed: ^c	399	321	78

Card & Kreuger (1994): Summary II



	Stores in:	
Variable	NJ	PA
1. Distribution of Store Types (perc	centages):	
a. Burger King	41.1	44.3
b. KFC	20.5	15.2
c. Roy Rogers	24.8	21.5
d. Wendy's	13.6	19.0
-		

Card & Kreuger (1994): Model

\$\$\widehat{Employment_{i t}}=\beta_0+\beta_1 \, NJ_{i}+\beta_2 \, After_{t}+\beta_3 \, (NJ_i \times After_t)\$\$

- PA Before: \(\beta_0\)
- PA After: \(\beta_0+\beta_2\)
- NJ Before: \(\beta_0+\beta_1\)
- NJ After: \(\beta_0+\beta_1+\beta_2+\beta_3\)
- **Diff-in-diff**: \((NJ_{after}-NJ_{before})-(PA_{after}-PA_{before})\)

	PA	NJ	Group Diff \((\Delta Y_i)\)
Before	\(\beta_0\)	\(\beta_0+\beta_1\)	\(\beta_1\)
After	\ (\beta_0+\beta_2\)	\ (\beta_0+\beta_1+\beta_2+\beta_3\)	\(\beta_1+\beta_3\)

Card & Kreuger (1994): Results



	Stores by state		
Variable	PA (i)	NJ (ii)	Difference, NJ – PA (iii)
 FTE employment before,	23.33	20.44	-2.89
all available observations	(1.35)	(0.51)	(1.44)
FTE employment after,	21.17	21.03	-0.14
all available observations	(0.94)	(0.52)	(1.07)
Change in mean FTE	- 2.16	0.59	2.76
employment	(1.25)	(0.54)	(1.36)